Challenges Related to Underground Transmission of Bulk Power Using HVAC

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***Abstract*—A brief discussion of design/construction differences between overhead and underground transmission is given. Following this, the fundamental electrical differences between overhead and underground HVAC transmission systems are outlined. It is shown why underground HVAC transmission lines have both hard and practical length limits while overhead lines do not. Further, a method for determining hard and practical length limits for underground HVAC lines is given along with an explanation about why the limits become shorter for higher voltages.**

***Index Terms*—underground power transmission lines, reactive power, power cables, capacitance**

I. Introduction

FOR many years, the issue of whether to use overhead or underground systems for transmitting electrical power using high voltage transmission lines has been discussed. Issues driving this range from aesthetics to audible noise to electromagnetic field exposure to susceptibility to storm damage. It has often been noted that lower voltage underground distribution networks are common in newer construction and that cities often have no option but to use high voltage underground systems to transmit bulk power. More recently, high voltage overhead transmission lines have been implicated in wildfire initiation. As a result, electrical utilities have been reviewing policies for deciding whether to use overhead or underground power transmission systems. Further, there is new discussion at the national level about a significant enhancement of the electrical transmission network to support increased electrification.

Given this, it is important to revisit issues relevant to using HVAC overhead or underground transmission. What are these issues? It is often stated that underground transmission lines are limited in length due to charging current. There is truth in this, but overhead lines also have (albeit smaller) charging currents but do not appear to be limited in length. Why? What then, are the essential differences between overhead and underground transmission? What is the basis for both hard and practical length limits for underground lines. Finally, why (and by how much) do the limits become smaller as the voltage is raised.

Here, issues related to both design/construction and operation of these systems will be considered. Emphasis is placed on challenges to the use of underground HVAC transmission lines to transmit bulk electrical power.

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II. Brief Summary- Design/Construction Issues

Numerous considerations must be taken into account when deciding between overhead and underground transmission systems. Here, the treatment of design/construction issues will be brief given 1) available information on this topic [1] and 2) that the main issue in this paper is electrical operation.

The environment in which a transmission line is to be placed may preclude (or at least make very difficult) either the installation of overhead or underground transmission. Long distances across deep water or restricted overhead space in urban areas may preclude the use of overhead transmission. Mountainous or hilly terrain or wetlands may preclude the use of underground transmission. Excavation for underground transmission can be disruptive and is not always easy.

The cost of constructing an underground transmission line varies from on the order of 4 to more than 10 times the cost of an overhead one. These costs include the more expensive underground cable compared to that of overhead conductors, as well as materials associated with burial in the earth (e.g., ducts enclosed in concrete) as well as that of continuous excavation for underground transmission versus poles or tower structures and foundations for overhead transmission.

Any transmission line conductor will have finite electrical resistance and carry a significant amount of electric current. This results in heating of conductors due to ohmic losses as well as hysteresis and eddy current losses in steel pipes (if used) Cooling of overhead conductors is via thermal radiation through the air and convection due to wind while cooling of underground conductors is via thermal conduction through the earth. Given these, the ampacity of overhead conductors is generally larger than for comparable underground cables.

Overhead transmission lines are subject to damage from lightning, tree falls, severe wind storms, excessive icing, and earthquakes. Lightning usually causes intermittent outages given the lightning protection systems. However, damage from other environmental stresses may cause transmission line failures that require maintenance. Underground transmission lines may be subject to lightning strikes, flooding, and earthquakes as well as damage from human digging.

Well-designed overhead transmission lines operating at voltages greater than 110 kV have low total outage rates of about 1 per 100 km of line per year [2]. Statistics show that underground transmission lines are more reliable than overhead lines [3], but the time and cost of fixing underground lines tends to be significantly larger than overhead lines [4]. While locating the specific site of a fault on overhead transmission lines is a fairly rapid process because the components are visible, finding faults on underground transmission lines can be much more difficult and time consuming due to the need for opening vaults and/or excavation before inspection. While outages on overhead lines usually last less than a day, the typical duration of an XLPE outage is 5 to 9 days.

Overhead transmission lines have been known to survive without replacement for nearly a hundred years. XLPE cables, had early problems associated with moisture ingress and higher temperatures. However, these have been corrected and XLPE systems installed in the late 1980's and early 2000's at respective voltages of 230 kV and 345 kV are still operating.

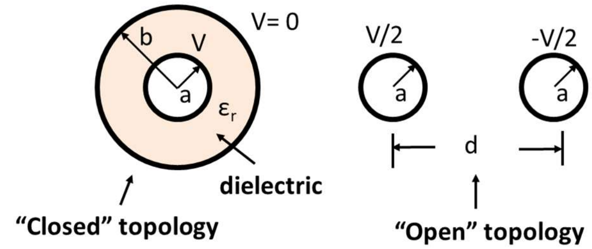
Above ground electric fields from underground cables effectively zero due to shielding by the cable neutral and other metallic layers, concrete, and soil. However, electric fields can be substantial in the vicinity of overhead transmission lines and must be managed according to utility policy or local regulations. Magnetic fields from underground transmission lines are not shielded by the earth but are generally lower than those of overhead transmission lines because the underground conductors are placed closer together. However, underground cable bury depth is generally less than 3 meters. Therefore, despite better cancellation effects, magnetic field exposure can be higher for conductors due to their proximity.

III. Issues related to Electrical operation - background

To begin, it will be assumed that three-phase transmission lines are balanced and that they can be analyzed on a "per-phase" basis. This will reduce the complexity of the analysis and provide more clarity. Hence, only a single phase of a three phase system will be shown and analyzed.

It can be shown that pure transverse electromagnetic (TEM) guided waves can be supported by a pair of infinitely long separate perfect conductors [5]. Two different topologies that satisfy this criteria are illustrated in Fig. 1. The first (Fig. 1a) is a "closed" topology that consists of two coaxial conductors of radius a and b separated by a dielectric material with relative dielectric constant ε_r that fills the space between them. In such a configuration, the TEM electric and magnetic fields are confined to the area between the conductors. This topology has been used for underground cables. The second (Fig. 1b) is an "open" topology consisting of two parallel cylindrical conductors of radius a separated by a distance d in free space. Configurations like this (or with additional wires) have been used historically for high voltage overhead transmission lines. Also shown in Fig. 1 are voltages to which the conductors are energized. One reason why TEM guided waves are important is that they are the only discrete guided "mode" that has a cutoff frequency of zero [5]. Hence this is the only guided wave mode available for propagation of low frequency voltages and currents.

One property of pure TEM waves is relevant here. If there is no reflected wave, (i.e., surge impedance loading) the energy stored in the electric and magnetic fields at any point in space and time is equal. Yet, the balance in energy components is compromised if there are both forward and reflected waves (i.e., the load is not equal to the surge impedance) which is the most common case for power lines. When this is true, the voltage and current at the input to the transmission line are no longer in phase. In power engineering terms, this means that reactive power input at either or both ends of the transmission line is non-zero. For constant voltage, this reactive power results in currents that are not related to the flow of real power but do contribute to the thermal losses in the system. Hence, they limit the transfer capacity and efficiency of the transmission system.



(a) (b)

Fig. 1 Two topologies that can support guided TEM waves.

A. *Quasi-TEM Guided Waves*

If the conductors are not perfect, then the guided wave still exists but is termed a "Quasi-TEM" wave which reduces to the pure TEM wave as the conductivity of the conductors approaches infinity. Here, if both the conductors and the dielectric are considered slightly lossy and if the effect of both the electric and magnetic fields dominates the loss, the waves will be considered Quasi-TEM. Most properties of pure TEM waves will remain approximately true.

B. *Representation in Terms of Distributed Parameters*

Using low frequency solutions to Maxwell's equations, it is often possible to derive expressions for propagation of quasi-TEM modes in terms of voltages between and current flow on the conductors and the distributed circuit parameters capacitance, inductance, resistance, and conductance per unit length [6]. Such an approach will be taken here. More specifically, the waves will be described by the one dimensional Telegrapher's equations given in (1) and (2) below in the frequency domain. The results are

$$-\frac{\partial \hat{V}(z)}{\partial z} = \left(r + j\omega l\right)\hat{I}(z) \qquad (1)$$

$$-\frac{\partial \hat{I}(z)}{\partial z} = \left(g + j\omega c\right)\hat{V}(z) \qquad (2)$$

Here, the voltage $\hat{V}(z)$ and current $\hat{I}(z)$ are functions of distance for either of the two configurations described in Fig. 1, ω is the radian frequency and the carat notation " ^ " indicates a phasor quantity. Time variation is assumed as $e^{j\omega t}$. The per unit length parameters in (1) and (2) are the inductance (l), capacitance (c), resistance of the conductors (r), and conductance of the dielectric (g). These can be related to the electric and magnetic fields between and in the conductors. Note, also, that the voltage and current can always be related to electromagnetic field variables. The voltage is defined as the negative line integral of the transverse electric field between the conductors and the current as the transverse magnetic field at the conductor surface times the circumference of the conductor.

If the derivative of (1) is taken with respect to z and (2) is then substituted into it, the result is

$$\frac{\partial^2 \hat{V}(z)}{\partial z^2} + \left(r + j\omega l\right)\left(g + j\omega c\right)\hat{V}(z) = 0 \qquad (3)$$

Solutions of (3) can be written as

$$\hat{V}(z) = A(\omega)e^{\mp j\gamma z} \quad (4)$$

where the minus (plus) sign indicates propagation in the positive (negative) direction and the propagation constant is

$$\gamma = \beta - j\alpha = \sqrt{-(r + j\omega l)(g + j\omega c)} \quad (5)$$

where β and α are respectively the phase and attenuation constants. If $r = g = 0$, then $\gamma = \beta = \omega\sqrt{lc}$, the phase velocity of the wave is $v_p = 1/\sqrt{lc}$ and there is no attenuation with z. If, however, r and/or g are not equal to zero, then the wave is attenuated resulting in ohmic loss and heating of conductors.

C. Differences Between the Two Topologies

To illustrate the differences between the two topologies the capacitance per unit length is given in (6) for the closed and in (7) for the open topology shown in Fig. 1.

$$c_{closed} = \frac{2\pi\varepsilon_0\varepsilon_r}{\ln(b/a)} \text{ Farads/m.} \quad (6)$$

$$c_{open} \cong \frac{\pi\varepsilon_0}{\ln(d/2a)}, \quad d >> a \text{ Farads/m} \quad (7)$$

where ε_0 is the permittivity of free space and ε_r is the relative dielectric constant of the homogeneous medium which separates the transmission line conductors. Substitution of typical overhead transmission line or cable geometric parameters and relative dielectric constants results in capacitance values on the order of 10 times smaller for the open wire case than for the dielectric filled coaxial cable.

The inductance per unit length (the small part that is associated with the magnetic field inside the conductors is neglected here) for the two topologies can be written as (8) for the closed and in (9) for the open topology of Fig. 1.

$$l_{closed} = \frac{\mu_0}{2\pi}\ln(b/a) \text{ Henries/m} \quad (8)$$

$$l_{open} \cong \frac{\mu_0}{\pi}\ln(d/2a), \quad d >> a \text{ Henries/m} \quad (9)$$

where μ_0 is the permeability of free space. It is useful to note that the inductance per unit length of the open topology is generally larger than that for the closed topology. This reversal is expected since the speed of light in the medium separating the two conductors is inversely proportional to the product of inductance and capacitance for each topology.

Another relevant parameter for these two topologies is their surge (or characteristic) impedances. Neglecting internal inductance of the wires, these are given in (10) for closed and (11) for open topology as

$$Z_{S(closed)} = \sqrt{\frac{l_{closed}}{c_{closed}}} = \sqrt{\frac{\mu_o}{\varepsilon_0\varepsilon_r}}\frac{\ln(b/a)}{2\pi} \text{ Ohms} \quad (10)$$

and

$$Z_{S(open)} = \sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{\ln(d/2a)}{\pi} \text{ Ohms} \quad (11)$$

For typical values of *b/a* and ε_r used for power cables, the characteristic impedance for the closed topology is on the order of 20 – 60 Ohms. For typical values of *d/2a* used for overhead transmission lines, the characteristic impedance for the open topology is on the order of 200 – 400 Ohms. Hence, the surge (characteristic) impedance of an open wire system is on the order of 10 times larger than that for a coaxial cable.

Some of the simple models used here involve loss, but this loss will be represented only by the resistance per unit length. Issues such as the internal inductance of wires and the conductance of the insulation will be ignored since they would only obscure the important points to be made.

As will be illustrated in the rest of this paper, there are numerous consequences due to the differences in the parameters defined above. Generally, they point to greater challenges for underground transmission than for overhead transmission systems.

IV. Issues related to Electrical Operation – circuit Approach

A. Technical Background

As mentioned above, all high voltage power lines have thermal limits on how much power they can safely transfer. For overhead lines, these limits have their origin in maximum temperatures of conductors that are derived using the type of conductor as well as a balance between ohmic/solar heating and cooling due to convection (wind) and thermal radiation. For underground lines, the balance is between ohmic, dielectric, and possibly other[1] losses and thermal conductivity of the materials in which the line is embedded. Conductors, insulation, and hardware such as splices that are too hot for long periods of time can either deteriorate or (for overhead lies) sag to levels that violate safety standards.

Consider first, a transmission line modelled as an *R, L, C* circuit that is open circuited (i.e., with no load) as shown in Fig. 2. Here $R = r\ell$, $L = l\ell$ and $C = c\ell$ where ℓ is the length of the transmission line. This derivation will be shown shortly to be valid for lengths less than about 100 km.

The current into the line can be written as

$$I = \frac{V_g}{r_{LOSS}\ell + j\left(\frac{\omega^2 lc\ell^2 - 1}{\omega c\ell}\right)} \cong \frac{V_g}{r_{LOSS}\ell - j\left(\frac{1}{\omega c\ell}\right)} \cong j\omega c\ell V_g \quad (12)$$

since at 60 Hz $\omega^2 lc\ell^2 << 1$ if $\ell <\sim 100$ km and generally, $r_{LOSS}\ell << j(1/\omega c\ell)$.

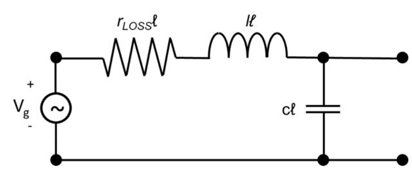


Fig. 2. Transmission line driven by a voltage source at the near end which is open circuited at the far end.

It is clear from (12) that the current into the transmission line (with no load) is proportional to the length of the line. Further, for a 345 kV underground cable system the charging current is on the order of 10 – 20 A/km. Hence, an energized open

[1] Heating due to hysteresis and eddy current losses in protective steel pipe can significantly reduce thermal ratings of underground cable.

circuited 10 km length of cable may have between 100 – 200 A of capacitive current. Since typical (single phase) cables have thermal limits of 800 – 1500 A, it is clear that, beyond some length $\ell_{\max}$, the capacitive current alone will exceed the thermal limit. $\ell_{\max}$ would then appear to represent a maximum length for uncompensated[1] cable. The source must absorb reactive power $= \mathrm{Im}(V_g I^*) = \omega c \ell V_g^2$ corresponding to time averaged energy exchange with the capacitor. The required current exceeds the rated current if $\ell > \ell_{\max}$.

However, this approach to describing line length limits is not sufficiently general as there is a problem extending it to the overhead case. If this issue prevents an underground cable from having a length greater than $\ell_{\max}$ (on the order of tens of km) then it would follow that an overhead line cannot have a length greater than about $10\ell_{\max}$ on the order of 100's of km because it has about 1/10th of the capacitance of an underground line as can be seen from a comparison of (6) and (7). However, operational experience indicates that no such limit applies to overhead line lengths. Of course, overhead lines generally have current ratings larger than underground lines due to more efficient cooling mechanisms which might increase this limit, but the mechanism of capacitive current consuming capacity is not what constrains length in the overhead case. Overall, however, the principle of limited length due to reactive power characteristic still holds for overhead lines, but a more general approach must be used to describe it.

A hint about how this dilemma can be resolved comes from examining the energy storage mechanisms. Note, first, that the time averaged energy stored in the capacitance (i.e., electric field) of the transmission line[2] is $c\ell V^2/2$. However, there is also time averaged energy stored in the inductor equal to

$$\frac{1}{2} l\ell I^2 = \frac{1}{2} l\ell \omega^2 c^2 \ell^2 V_g^2 << \frac{1}{2} c\ell V_g^2 \tag{13}$$

The last inequality is derived from the assumption above that the transmission line was short (i.e., $\omega^2 lc\ell^2 << 1$). While the energy in the inductor is much smaller than the energy in the capacitance, it subtracts from the total reactive power the generator needs to absorb and hence reduces the total input current. This begs the question about whether (or not) connecting a load to the end of the transmission line (which increases the inductor current and stored energy) can reduce the need for reactive power enough that the total input current (due to both real and reactive power) is reduced.

To answer this question, consider the same transmission line, but with a load R_{LOAD} that absorbs power as shown in Fig. 3.

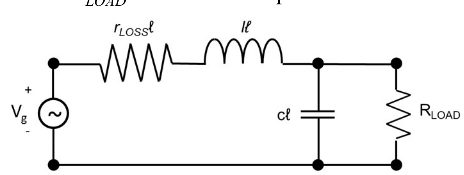


Fig. 3 Transmission line driven by a voltage source at the near end which is loaded by R_{LOAD} at the far end.

In this case, the total input current into the transmission line is

$$I = \frac{V_g\left(1 + j\omega c\ell R_{LOAD}\right)}{r_{LOSS}\ell + (1-\omega^2 lc\ell^2)R_{LOAD} + j\omega l\ell + j\omega c\ell R_{LOAD} r_{LOSS}\ell}$$
$$\cong \frac{V_g\left(1 + j\omega c\ell R_{LOAD}\right)}{\left(r_{LOSS}\ell + R_{LOAD}\right)\left[1 + j\omega\left(\frac{l\ell + c\ell R_{LOAD} r_{LOSS}\ell}{r_{LOSS}\ell + R_{LOAD}}\right)\right]} \tag{14}$$

Note again that $\omega^2 lc\ell^2 << 1$ if $\ell < 100km$. This current is purely real (which means that the average capacitor and inductor stored energies are equal) if

$$\frac{l\ell + c\ell R_{LOAD} r_{LOSS}\ell}{r_{LOSS}\ell + R_{LOAD}} = c\ell R_{LOAD}. \tag{15}$$

This condition is satisfied if

$$R_{LOAD} = \sqrt{\frac{l}{c}} = \sqrt{\frac{L}{C}} = Z_S \tag{16}$$

where, again, Z_S is the surge impedance and the transmission line is "surge impedance" loaded. Given this assumption, (14) becomes simply

$$I = \frac{V_g}{R_{LOSS} + R_{LOAD}} \cong \frac{V_g}{R_{LOAD}} \tag{17}$$

which is purely real. Note that the current is "in phase" with the voltage, hence no reactive power is needed to be supplied or absorbed by the generator. Energy is exchanged, but the average energy in the capacitor is the same as in that in the inductor. Therefore, energy exchange is entirely between the electric and magnetic fields of the line itself.

Now, this appears to be an excellent solution to reducing the need to supply or absorb transmission line reactive power. However, it leads to another problem which points to the importance of the difference between surge impedance for overhead and underground transmission lines.

B. Overhead vs. Underground Lines - Surge Impedance Loading

For a 345 kV overhead transmission line energized to 199 kV (line to ground) with a 250 Ohm surge impedance, the current in (17) becomes approximately 800 amps and the power and the (single phase) surge impedance loading is 159 MW. Given the natural cooling mechanisms associated with overhead transmission lines, this is a reasonable current which does not violate thermal limits. Further, this example has been validated by long experience with overhead transmission lines. Hence, it is reasonable to expect that overhead lines can be operated with comparatively little need for a source to supply or absorb reactive power[3].

For a 345 kV underground transmission line energized to 199 kV (line to ground), however, the surge impedance is much smaller; generally around 40 Ohms. In this case, the current in (17) becomes approximately 5000 amps. This is significantly larger than thermal limits will allow for typical underground lines. Hence, the operation of these lines at thermally acceptable

[1] Compensation would be in the form of a separate supply for capacitive current

[2] In this approximation, the Ferranti effect (increase in voltage at the end of the transmission line) is ignored

[3] Shunt reactors are still common for EHV systems to manage voltage during light load conditions when flows are below the surge impedance loading. Likewise, in long, heavily loaded overhead lines, series capacitors are sometimes used to augment the internal supply of reactive power.

current levels still results in a significant excess of reactive power and would require reactive compensation, even sometimes for relatively short lines.

The bottom line, then, is that the reduced surge impedance coupled with reduced thermal limits of underground lines results in an AC length limit that is difficult to overcome. In the next section, this issue will be revisited in a way that can be used to derive both hard and practical length limits.

V. Issues Related to Electrical Operation – Distributed Parameter Approach.

A. *Introduction*

In this section, the analysis of Section IV using circuit analysis as its basis will be generalized to the case for transmission lines of arbitrary length using a distributed parameter model for the transmission line. One additional advantage of this model is that the issue of stability limitations on power flow capacity (i.e., loadability) can be introduced.

B. *Technical Background*

In addition to thermal limits, the power capacity of HVAC transmission lines is limited by system stability considerations. To understand this, consider the simple power system shown in Fig. 4 that consists of one generator of known phasor voltage $\hat{V}_{g1}$ connected through a transmission line of arbitrary length modeled as a pi network with admittances (that are a function of transmission line length) Y_{gg} ,$Y_{g\ell}$ and $Y_{\ell\ell}$ to a load which requires a real power of P_ℓ MW and a second generator of known phasor voltage $\hat{V}_{g2}$. It is assumed here that the load requires more power than Generator 2 can deliver. Hence, the connection to Generator 1 through the transmission line.

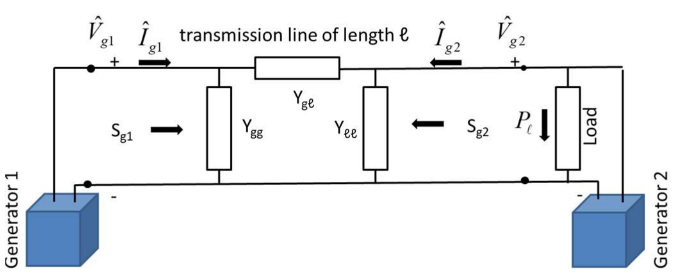


Fig. 4. Simple power system used to study power flow on arbitrary length transmission lines

The transmission line is characterized by the parameters

$$Y_{gl} = jY_S / \sin(\gamma\ell) \qquad (18)$$

and

$$Y_{gg} = Y_{ll} = -jY_S \tan(\gamma\ell/2) \qquad (19)$$

where $\gamma = \beta - j\alpha$, ℓ and Y_S are respectively the complex propagation constant, length, and characteristic admittance equal to $1/Z_S$ where Z_S is its characteristic (or surge) impedance of the transmission line.

It can be shown that the complex power supplied by Generator 1 is

$$S_{g1} = P_{g1} + jQ_{g1} = \hat{V}_{g1}\hat{I}^*_{g1} = \left(Y^*_{gg} + Y^*_{g\ell}\right)\left|\hat{V}_{g1}\right|^2 - Y^*_{g\ell}\hat{V}_{g1}\hat{V}^*_{g2} \qquad (20)$$

where P_{g1} and Q_{g1} are, respectively, the real and reactive power supplied by Generator 1. If the transmission line is lossless (i.e., $\alpha = 0$) and $\left|\hat{V}_{G1}\right| = \left|\hat{V}_{G2}\right|$, then the complex power flow from Generator 1 to Generator 2 is

$$S_{g1} = P_{g1} + jQ_{g1}$$
$$= P_{SIL}\left\{\frac{\sin(\theta_{g1} - \theta_{g2})}{\sin(\beta\ell)} + j\frac{\left[-\cos(\beta\ell) + \cos(\theta_{g1} - \theta_{g2})\right]}{\sin(\beta\ell)}\right\} \qquad (21)$$

where $\theta_{g1} - \theta_{g2}$ is the difference in the phase angles of the two generators, $\beta = 2\pi/\lambda$ where λ is the wavelength (5000 km in air at 60 Hz) and ℓ is the transmission line length. P_{SIL} is the surge impedance load equal to

$$P_{SIL} = Y_S\left|\hat{V}_{g1}\right|^2 = \frac{\left|\hat{V}_{g1}\right|^2}{Z_S} \qquad (22)$$

where V_{g1} (RMS) is the voltage magnitude of generator 1. It is useful to note here that 1) if the two voltage magnitudes are again assumed to be equal, 2) if the line is assumed to be lossless, and 3) if there are no reflections from the load and generator 2, then the voltage along the line is simply a forward traveling wave:

$$V(\ell) = V_{g1}e^{-j\beta\ell}\,. \qquad (23)$$

This is the case for surge impedance loading. If, based on (23), $\theta_{g1} - \theta_{g2} = +\beta\ell$ is inserted into (21)

$$S_{g1} = +Y_S\left|\hat{V}_{g1}\right|^2 \qquad (24)$$

This is consistent with the result in Section IV; the reactive power supplied by Generator 1 is zero. This means that the capacitive reactive power supplied by the transmission line along its length is exactly cancelled by the inductive reactive power consumed by the transmission line along its length. Again, this is characteristic of transmission lines that are loaded with their surge impedance.

If Q_{g1} is negative and large compared to P_{g1}, such as might happen when the power required at the load is relatively low, then the generator must consume significant reactive power. Alternatively, the excess reactive power could be consumed by a passive device such as a shunt reactor.

The "total power" (i.e., magnitude of the real plus reactive power) into the transmission line could be expressed as a fraction of P_{SIL} as

$$\left|\frac{S_{g1}}{P_{SIL}}\right| = \left|\frac{\sin(\theta_{g1} - \theta_{g2}) + j\left[-\cos(\beta\ell) + \cos(\theta_{g1} - \theta_{g2})\right]}{\sin(\beta\ell)}\right| \qquad (25)$$

The input current (again normalized by P_{SIL}) could be obtained by dividing (25) by V_{g1}.

C. *Overhead Lines*

For short overhead lines (i.e., $\beta\ell = 2\pi\ell/\lambda << 1$), it is clear that P_{g1} in (21) can exceed P_{SIL} by a significant amount by increasing $\theta_{g1} - \theta_{g2}$ in (21) to be larger than $\beta\ell$ but still small compared to 1. However, for these short lines the power transfer capability is constrained by thermal limits as discussed above and illustrated in Fig. 5. Note here that the length of the transmission line is plotted in terms of a fraction of a wavelength (λ = 300,000/f km). This is done so that later the effect of changing frequency can be more readily discussed. For longer lines (for which $\beta\ell = 2\pi\ell/\lambda$ is comparable to 1 or larger), $\theta_{g1} - \theta_{g2}$ must be considerably larger in order to even reach P_{SIL}. However, (and this is where the "stability" part comes in) $\theta_{g1} - \theta_{g2}$ should not exceed 40 to 50 degrees in order to prevent a loss of synchronization between the generators during system disturbances [7]. Thus, for longer transmission lines, the maximum power carrying capacity of the line is generally somewhat less than P_{SIL} (often called the surge impedance loading limit). This is also illustrated in Fig. 5.

The bottom line is that overhead lines of arbitrary length can be operated at (or near) their stability limit without the penalty of excessive need for reactive power supplied from the generator. If, however, the load is (for example) small at some point in time, some additional shunt compensation may be needed to absorb the reactive power supplied by the line.

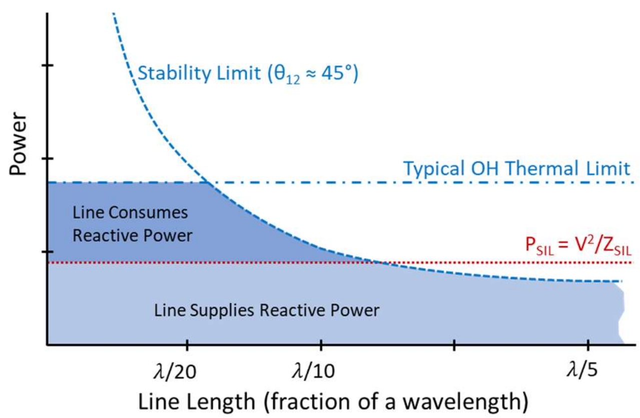


Fig. 5. Overhead Transmission Line Power Limits Based on Thermal and Stability Considerations. Operation within shaded regions is compliant with thermal and stability limits.

As a final comment, it should be noted that for longer line lengths, the surge impedance limited power flow is a fair approximation to the power capacity of the line. This is evident from an examination of Fig. 5 (i.e., for longer line lengths $P_{maxL} \approx 1$).

D. Underground Lines

As for overhead transmission lines, there are thermal limits on the capacity of underground transmission lines to carry power. However, these limits are different from those for overhead lines and (generally) smaller. This is illustrated in Fig. 6 which should be compared to the limit in Fig. 5 for overhead lines. While there is no solar radiation input, the cooling mechanisms for underground transmission lines are much more limited. More specifically, there is no convective cooling due to wind. Rather, cooling is due to heat conduction in the soil away from the transmission line and this depends on the thermal resistivity of the material in which the cable is buried as well as the ground and ambient temperatures.

Another significant characteristic for underground lines is that the stability limit is much higher than for overhead lines since Z_S is much smaller for underground transmission (see (22)) and P_{SIL} is much larger. Hence, underground transmission lines will generally not be stability limited. Given the fact that the surge impedance loading limit is so much higher than the thermal limit, it is likely that the reactive power supplied by underground lines will be significantly larger than overhead lines. By implication, this means that there is likely to be a severe length limit for underground lines, particularly if the conditions of the interconnected system are such that the excess reactive power at all points along the line flows in the same direction. In summary, underground lines cannot generally be of arbitrary length because the difference between surge impedance and thermal limits prevent them from being operated without a significant need for reactive power supplied from the generator. Some additional explanation about this is warranted.

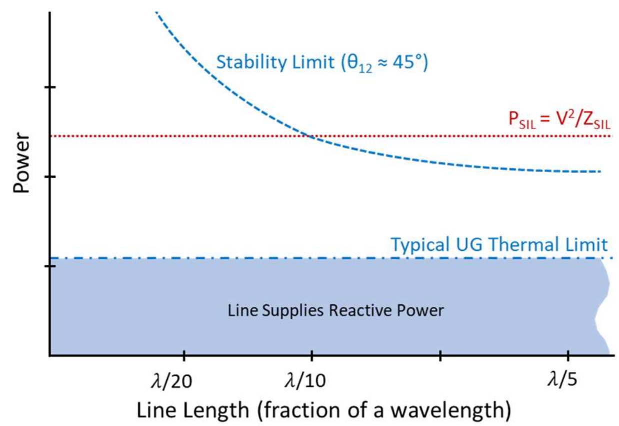


Fig. 6. Typical Underground Transmission Line Power Limits Based on Thermal and Stability Considerations. Here the thermal limit is significantly less than the surge impedance loading limit.

If the rated thermal total power limit for the underground line is known as a percentage of underground surge impedance loading, P_{SIL}^{UG} (e.g., 50% P_{SIL}^{UG}), then a contour plot of (25) can be used to identify conditions for which the cable is loaded beyond its capacity. Here the thermal limit will be identified as

$$P_{ThermLim}^{UG} = \eta P_{SIL}^{UG} \quad (26)$$

where η is the fraction of P_{SIL}^{UG} that is equal to the underground thermal limit. In Fig. 7, contours of constant total power (expressed as percentages of the surge impedance limit) are plotted as a function of both line length and generator phase difference. These can be compared to the thermal limit in (26) for any value of η, again expressed as a percentage of the surge impedance limit.

One well-known characteristic of cables can easily be discerned from this graph. For zero real power transferred (i.e., generator phase difference equal to zero) it is clear that the input total power (in this case all reactive power) increases linearly as the line length increases. This is consistent with the idea that the capacitive current is larger for longer line lengths when there is no real power transfer (e.g., no load and no voltage difference between the generators) as shown in (21). In addition, if the thermal limit for a given underground system is known as a fraction of the surge impedance limit, then the region of Fig. 7 for which the system can be operated without violating the thermal limit can be identified. For example, if the thermal limit is 20% of the surge impedance limit, the only portion of the plot which corresponds to allowable operation is to the left of the 20% contour in Fig. 7.

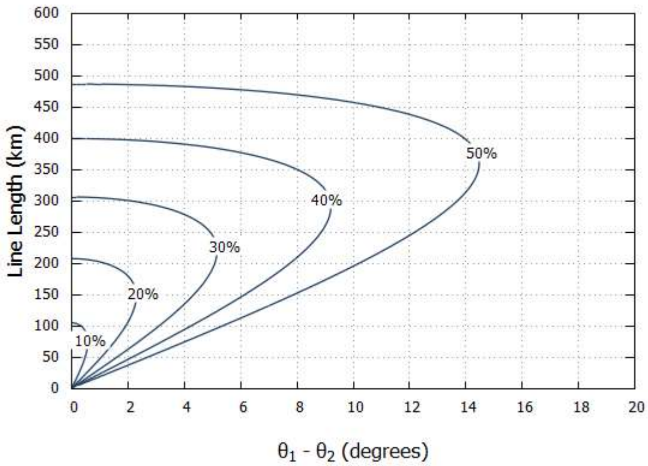


Fig. 7. Contour plot of total power into the underground transmission line of Fig. 4 relative to surge impedance loading as a function of line length and generator phase angle difference. f = 60 Hz while a = 1.67 cm, b = 4.89 cm, ε_r = 2.3, and Z_s = 42.5 Ohms. The numbers on the graph curves represent the total power expressed as a percentage of the surge impedance loading limit.

A complement to Fig. 7 can be found by plotting the real power flowing from Generator 1 toward the load in Fig. 4. The result for contours of constant real power (expressed as percentages of PSIL) are given in Fig. 8.

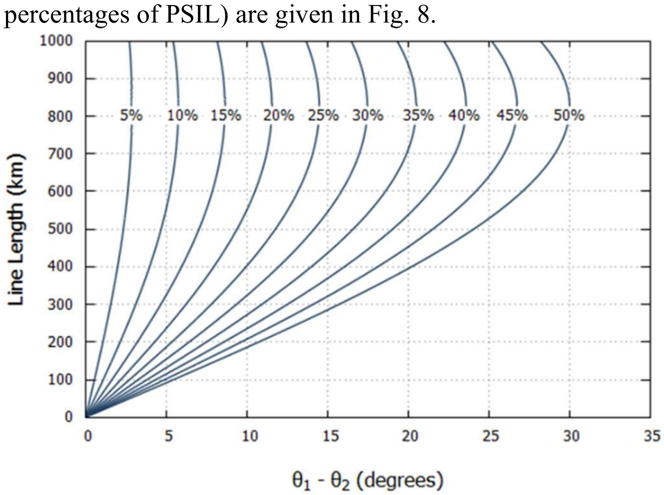


Fig. 8. Contour plot of real power through the underground transmission line of Fig. 4 as a function of both line length and generator phase difference. All parameters are the same as in Fig. 7. The numbers on the graph represent the real power from Generator 1 expressed as a percentage of the surge impedance limit.

Fig. 7 and Fig. 8 can now be combined into one plot in order to evaluate the performance of long underground cables. The result of this is shown in Fig. 9.

345 kV Example

Consider, the case for a typical 345 kV underground transmission line (single phase to ground voltage of 199 kV) using 1500 kcmil cable which has a (single phase) thermal limit of approximately 197 MVA. For a cable with dimensions a = 1.67 cm, b = 4.89 cm and a dielectric constant ε_r = 2.3, the surge impedance is 42.5 Ohms, and the surge impedance power limit is 932 MW. Hence η = 0.21 and

$$P^{UG}_{ThermLim} = 0.21 P^{UG}_{SIL} \tag{27}$$

As mentioned earlier, the only acceptable operating region is to the left of the 20% line in Fig. 7 (also shown in Fig. 9). First, transmission lines longer than about 200 km cannot be used under any conditions[1]. Second, comparing the solid (total power) and dashed (real power) lines for the 20% case in Fig. 9, it can be seen that over the range of cable lengths from 0 to approximately 80 km, real power dominates reactive power and cables can be used to transfer real power up to the thermal limit (i.e., 20% of P^{UG}_{SIL}). However, because reactive power must be supplied by the generators for longer cables, the real power transfer is more limited in this case. For example, 150 km and 175 km cable lengths would be limited to approximately 15% and 10% of P^{UG}_{SIL} respectively. This represents a serious limitation on long uncompensated lengths of cable. Hence the practical length limit for this example is approximately 80 km assuming reactive power is absorbed by generators or compensation at both ends.

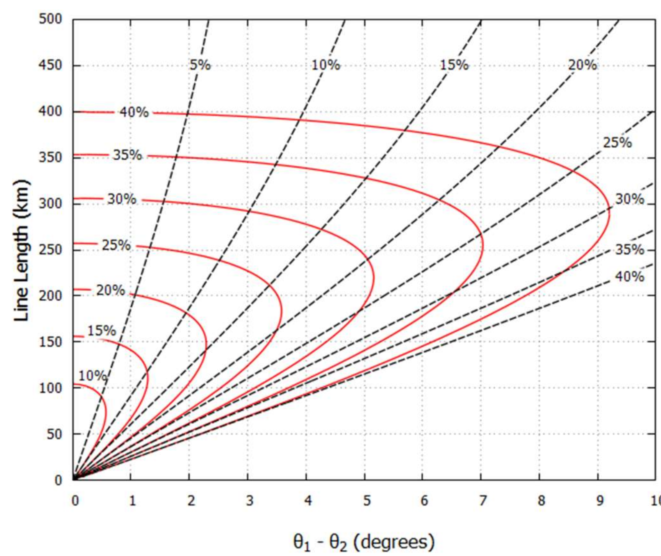


Fig. 9. Contour plots of Figs. 7 and 8 superimposed for the underground transmission line of Fig. 4 as a function of both line length and real power transmission. All parameters are the same as in Fig. 7. The numbers on the graph represent the total (solid lines that curve back on themselves) and real (dashed lines which extend to infinity) power expressed as a percentage of the surge impedance limit.

115 kV Example

Consider the case for a typical 115 kV underground transmission line (single phase to ground voltage of 66 kV) using 1500 kcmil cable which has a (single phase) thermal limit of approximately 70 MVA. For a cable with dimensions a = 1.67 cm, b = 3.11 cm and a dielectric constant ε_r = 2.3, the surge impedance is 24.6 Ohms, and the surge impedance power limit is 177 MW. Hence η = 0.40 and

[1] Here, reactive power is supplied from both ends of the cable. If a generator is connected to only one end of the transmission line, then the maximum length would be only 100 km, half that shown in Fig. 9.

$$P_{ThermLim}^{UG} = 0.4 P_{SIL}^{UG} \quad (28)$$

or the thermal limit is 40% of the surge impedance limit.

As mentioned earlier, the only acceptable operating region is to the left of the 40% line in Fig. 9. First, transmission lines longer than 400 km cannot be used under any conditions. Second, over the range of cable lengths from 0 to approximately 160 km, real power dominates reactive power and cables can be used to transfer real power up to the thermal limit (i.e., 40% of P_{SIL}^{UG}). However, because reactive power must be supplied by the generators for longer cables, the real power transfer is more limited. For example, 280 km and 460 km cable lengths would be limited to approximately 30% and 15% of P_{SIL}^{UG} respectively. Hence the practical length limit is approximately 160 km.

VI. Discussion

At a given voltage level, underground power lines can transmit roughly as much power as overhead transmission lines. However, uncompensated underground transmission lines are constrained in length due to the excess reactive power supplied by cable capacitance. It is also clear from the above examples, that lower voltage underground lines can be used to effectively transmit power over longer lengths than higher voltage lines. However, the amount of power that can be transmitted is still lower for lower voltage lines and it is difficult to see how they could be used to replace higher voltage overhead transmission lines that can carry significantly higher power. This tradeoff suggests an optimization problem where, for a given length, maximum practical power transfer may not always correspond to the highest voltage cable.

One corollary is that lower voltage underground distribution lines are less length constrained due their lower reactive power production. Also, the distances for which distribution lines are used are well within length limits for lower voltage lines and the power required to be carried is smaller.

VII. What can be done to resolve the issues raised here?

A cooling system could be used to raise the thermal limits of underground lines. However, this would require additional infrastructure and adds substantially to the cost of underground transmission lines.

Periodic (on the order of 10's of km) reactive compensation could be used to prevent exceeding thermal limits. This can be done using shunt reactors although the amount of compensation needed would depend on the load required. Further, the use of shunt reactors can cause other operational issues such as unintended resonances. High voltage shunt reactors are costly, require a small substation, and present switching challenges which require special-purpose circuit breakers designed to interrupt inductive currents.

HVDC could be used since there is no reactive power at DC. Note also from Figs. 5 and 6 that the length of any transmission line as a fraction of wavelength becomes zero in this case. In fact, HVDC transmission lines are used exclusively for long undersea connections for which no AC system would be possible. However, DC transmission lines require both special cable and costly AC/DC converter terminals at each end.

VIII. Conclusions

1. Replacing overhead with underground high voltage transmission lines presents major challenges both in terms of cost and fundamental physics.
2. There are fundamental differences between underground and overhead high voltage transmission lines due to their topology and the relationship between surge impedance loading and thermal loading limits.
3. Hard length limits exist for underground but not overhead high voltage transmission lines because overhead lines can be operated at or near surge impedance loading.
4. Practical length limits exist for underground lines that are shorter than hard limits. The use of lengths greater than practical limits results in reduced capacity for real power flow.
5. Both hard and practical length limits are longer for transmission lines at lower voltage.

IX. References

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X. Biographies

Robert G. Olsen (S'66–F'92–LF'11) received the B.S.E.E. degree from Rutgers University, New Brunswick, NJ, USA, in 1968, and the M.S. and Ph.D. degrees from the University of Colorado, Boulder, CO, USA, in 1970 and 1974, respectively. Since 1973, he has been with Washington State University, Pullman, WA, USA. Other positions include Senior Scientist with Westinghouse Georesearch Laboratory, NSF Faculty Fellow with GTE Laboratories, and Visiting Scientist with ABB Corporate Research and EPRI. His research interest is application of electromagnetic theory to high-voltage transmission systems. He has nearly 85 publications in refereed journals. He is one of the authors of the AC Transmission Line Reference Book—200 kV and Above (EPRI, 2005). Dr. Olsen is an Honorary Life Member of the IEEE Electromagnetic Compatibility (EMC) Society. He is also past USNC representative to CIGRE Study Committee 36 (EMC) and Past Chair of the IEEE Power Engineering Society AC Fields and Corona Effects Working Groups. In addition, he is also the Past Associate Editor for the IEEE Transactions on Electromagnetic Compatibility and Radio Science.

Jon T. leman (S'01–M'01–SM'17) received the B.S.E.E. and M.S.E.E degrees from the University of Idaho, Moscow, ID, USA, in 2001 and 2010, respectively. He completed his Ph.D. at Washington State University, Pullman, WA, USA in 2021. Dr. Leman is currently a principal engineer with POWER Engineers, Inc. and co-owner of the software company Electric Utility Design Tools, LLC. He has over 20 years of experience in power system engineering with emphasis in overhead and underground transmission studies. His research interests include power system electromagnetics, numerical methods, transmission planning, and transient analysis. Dr. Leman is a member of CIGRE and is a registered professional engineer in the state of Idaho.